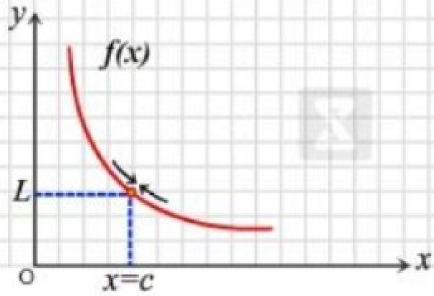
Limit

Existence of a Limit



$\lim_{x \to c} f(x) = L$ iff

$\lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x)$

Properties

If $\lim_{x \to c} f(x)$ & $\lim_{x \to c} g(x)$ exist

Scalar Multiple	$\lim_{x \to c} [b.f(x)] = b.\lim_{x \to c} f(x)$
Sum or Difference	$\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$
Product	$\lim_{x \to c} [f(x) . g(x)] = \lim_{x \to c} f(x) . \lim_{x \to c} g(x)$
Quotient	$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} f(x) / \lim_{x \to c} g(x)$
Power	$\lim_{x \to c} [f(x)^n] = [\lim_{x \to c} f(x)]^n$ for all $n \in N$

Trigonometric Functions

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 = \lim_{x \to 0} \frac{x}{\sin x} = \lim_{x \to 0} x \operatorname{cosec} x = \lim_{x \to 0} \frac{\sin^{-1} x}{x} = \lim_{x \to 0} \frac{x}{\sin^{-1} x}$$

$$\lim_{x \to 0} \frac{\tan x}{x} = 1 = \lim_{x \to 0} \frac{x}{\tan x} = \lim_{x \to 0} x \cot x = \lim_{x \to 0} \frac{\tan^{-1} x}{x} = \lim_{x \to 0} \frac{x}{\tan^{-1} x}$$

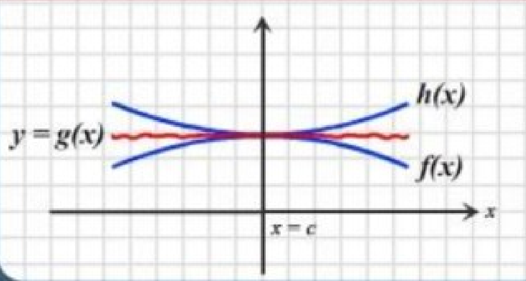
Exponential Functions

$\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a \; ; (a > 0)$	$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$
$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$	$\lim_{x \to 0} (1+x)^{1/x} = e$

Generalised Formula For 1^∞

Let $\lim_{x \to c} f(x) = 1$ & $\lim_{x \to c} \phi(x) \to \infty$ then $\lim_{x \to c} [f(x)]^{\phi(x)} = e^{\lim_{x \to c} \phi(x) \ln [f(x)]}$

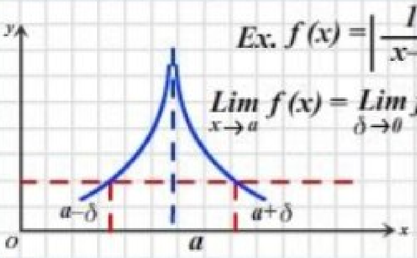
Sandwich / Squeeze Theorem



If f, g and h are three functions such that $f(x) \le g(x) \le h(x)$ for all x in some interval containing the point x = c and if

$\lim_{x \to C} f(x) = \lim_{x \to C} h(x) = L$ then $\lim_{x \to C} g(x) = L$

Limits of Infinity Theorems



Ex. $f(x) = \left| \frac{1}{x-a} \right|$

$\lim_{x \to a} f(x) = \lim_{\delta \to 0} f(x-a) = \infty$

If 'a' is a real number and 'r' is a positive rational number

$\lim_{x \to \infty} \frac{a}{x^r} = 0$; $\lim_{x \to -\infty} \frac{a}{x^r} = 0$